Angelic Content

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MLE Reading Group
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State Semantics

State Space: $\mathcal{S} = \langle S, \sqsubseteq \rangle$ is a state space, were S is a set of states, and $\sqsubseteq$ is a partial order on S.

Upper Bound: $s \in S$ is an *upper bound* of $T \subseteq S$ *iff* $t \sqsubseteq s$ whenever $t \in T$.

Least Upper Bound: $\bigsqcup T \in S$ is the (unique) *least upper bound* of $T \subseteq S$ *iff* $\bigsqcup T \sqsubseteq s$ whenever s is an upper bound of T.

$\mathcal{S}$-Completeness: Any subset of $T \subseteq S$ has a least upper bound $\bigsqcup T \in S$.

Complete Closure: T^* is the *complete closure* of $T \subseteq S$ *iff*: (i) $T \subseteq T^*$; and (ii) $\bigsqcup T' \in T^*$, for every subset $T' \subseteq T$.

Model: $\mathcal{M} = \langle S, \sqsubseteq, |\cdot| \rangle$, where $|\cdot|$ maps each sentence-letter p to a corresponding ordered pair $\langle |p|^+, |p|^- \rangle$ consisting of the exact verifiers and exact falsifiers for p, respectively.

Exact Verification: We define $\Vdash$ recursively, given the verifiers and falsifiers for each sentence letter of the language.

- We have a choice about whether or not $\vee^+$ and $\wedge^-$ are to be inclusive or non-inclusive.

Exact Verifiers: Let $|A| = \{s \in S : s \Vdash_{ni} A\}$ where $\Vdash_{ni}$ is non-inclusive.

Complete Verifiers: Let $\lceil A \rceil = \{s \in S : s \Vdash_{in} A\}$ where $\Vdash_{in}$ is inclusive.

$\mathcal{M}$-Completeness: A model $\mathcal{M}$ is *complete iff* $|p|^+$ and $|p|^-$ are complete for each sentence-letter p (see **Lemma 6**).

Question: Is the intended model $\mathcal{M}_I$ complete? (Validity is not of primary interest in speaking about the content of a given collection of sentences.)

Convexity and Containment

Subsumption: $T \sqsupseteq U$ *iff* $\forall(t \in T)\exists(u \in U)[u \sqsubseteq t]$.

Above: T is *above* U *iff* $f\colon T \to U$ such that $\forall(t \in T)[f(t) \sqsubseteq t]$.

English: T does not include elements that are irrelevant to U, i.e., elements that do not contain some $u \in U$ as a part.

Subservience: $U \sqsubseteq T$ *iff* $\forall(u \in U)\exists(t \in T)[u \sqsubseteq t]$.

Below: U is *below* T *iff* $g\colon U \to T$ such that $\forall(u \in U)[u \sqsubseteq g(u)]$.

English: U does not include elements that are irrelevant to T, i.e., elements that are not part of any $t \in T$.

—Example—

T:	$a \sqcup b, c, c \sqcup d$	e
U:	$a, c, c \sqcup d$	f

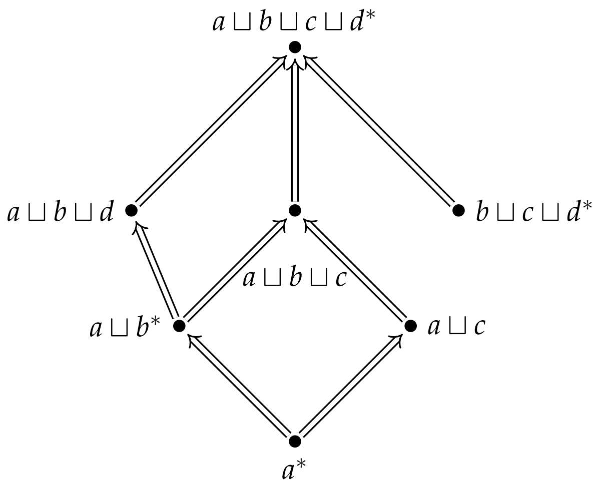
Containment: $T > U$ *iff* $T \sqsupseteq U$ and $U \sqsubseteq T$.

Convexity: T is *convex iff* $u \in T$ whenever $s, t \in T$ and $s \sqsubseteq u \sqsubseteq t$.

Convex Closure: Let T_* be the smallest convex set where $T \subseteq T_*$. ($T_* = \{u \in S : t \sqsubseteq u \sqsubseteq s,$ for some $t, s \in T\}$).

Span: Let $[T, s] = \{u : t \sqsubseteq u \sqsubseteq s$ for some $t \in T\}$.

Example: Let $T = \{a, (a \sqcup b), (b \sqcup c \sqcup d), (a \sqcup b \sqcup c \sqcup d)\}$.



$[T, (a \sqcup b \sqcup c \sqcup d)] =$
$\{a, (a \sqcup c), (a \sqcup b), (a \sqcup b \sqcup d), (a \sqcup b \sqcup c), (b \sqcup c \sqcup d), (a \sqcup b \sqcup c \sqcup d)\}$.

Lemma 7: When T is complete, $T_* = [T, \bigsqcup T]$ and is itself complete.

Lemma 8: $T > U$ *iff* $T_* > U_*$.

Lemma 9: If T and U are convex, then $T = U$ if $T > U$ and $U > T$.

Question: The argument following **Lemma 9** goes: (p) If $U > T$ is genuinely to represent T being *part* of the content U, then we would expect the relation to be antisymmetric. (c) Thus what these results show is that the notion of content with respect to which $>$ is a relation of partial content is the notion of convex containment. But how does (c) follow from (p)?

- Engineering: we want an anti-symmetric relation for partial content, and that is what convex containment gives us. So we should identify convex containment as the partial content relation we are looking for.
- But: it would seem better to be able to prove the converse of **Lemma 9**. This would show that if $>$ is anti-symmetric for some T and U, then T and U are convex, giving us a tighter connection (an equivalence) between partial content and convex containment.

Conclusion: "In particular, the exact semantics from the previous sections provides more information than is strictly necessary for determining partial content; it is the convex closure of the set of exact verifiers, rather than the set of exact verifiers itself, that should be taken to constitute the content of a statement for the purposes of ascertaining whether one statement is analytically entailed by another."

Answer: Presumably this means that, yes, the intended model is complete?

Subject-Matter

Replete Verifiers: Let $[A] = \lceil A \rceil_*$.

Subject-Matter$_{df}$: When $|A|$ is complete, $\sigma^+(A) = \bigsqcup(|A|) = \bigsqcup(\lceil A \rceil)$.

Subject-Matter$_R$: We define σ recursively, given the verifiers and falsifiers for each sentence letter of the language.

Lemma 10: $\sigma^+(A) = \bigsqcup(|A|)$ and $\sigma^-(A) = \bigsqcup(|\neg A|)$.

Bi-Latteral SM: Let $\sigma(A) = \sigma^+(A) \sqcup \sigma^-(A)$.

Theorem 11: $[A] = [|A|, \sigma^+(A)]$ and $[\neg A] = [|A|^-, \sigma^-(A)]$.

Analytic Equivalence: $\vDash_{\mathcal{M}} A \leftrightarrow B$ *if* $\vDash_{\mathcal{M}} [A] = [B]$.

English: $A \leftrightarrow B$ will hold in $\mathcal{M}$ if (i) every verifier of A contains and is contained in a verifier of B and (ii) every verifier of B contains and is contained in a verifier of A.